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THERMOVISCOELASTIC FINITE ELEMENT STRESS ANALYSIS BY USING ENDOCHRONIC APPROACH

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ABSTRACT

The purpose of this paper is to develop an incremental thermoviscoelastic finite element stress analysis method. The formulation is based on the following assumptions that (1) valanis' type endochronic hereditary integral isotropic constitutive equations with a viscoelastic intrinsic time measure are used, (2) the material is thermorheological simple so that temperature effect on material is embedded in reduced time, (3) shear relaxation modulus is composed of the first two terms of prony series, (4) the material is dilatational elastic that bulk modulus and thermal expansion coefficient are both constants the Leibnitz rule is used to derive the incremental thermoviscoelastic stress-strain equations, then the incremental governing equilibrium equations with a temperature-dependent viscoelastic pseudo force are derived by principle of virtual work.

Two examples are adopted to demonstrate the validity of the present analysis model, namly, creep of infinite strip in nonuniform temperature field across the width and a transient nonhomogeneous thermal stress analysis of solid propellant grain structure. From the good results of the above two examples, it is concluded the proposed method is rather satisfactory.

INTRODUCTION

Temperature effects for sensitive rate-dependent viscoelastic materials have to be considered in the presence of appreciable temperature variation. It is reasonable for moderate temperature field to assume the material is linear viscoelastic and thermorheological simple, therefore the effect of temperature on the material response functions can be accounted by reduced time through the Time-Temperature Shift Factor. Analytic solutions using Alfrey's [1] integral transform approach are limited to certain special problems [2,3]. Morland and Lee [4] point out the correspondence principle fails to hold for nonhomogeneous, transient temperature distribution. For analysis of viscoelastic

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structure with involved geometry and temperature distribution, various numerical solutions using the finite element method have been proposed [5-10].

In the present investigation the incremental endochronic thermoviscoelastic constitutive equations are derived first, then the incremental governing equilibrium equations are derived by principle of virtual work.

A computer program based on above formulations is written to analyze the following two examples: The first example is 1-D time hardening thermal creep analysis of an infinite plate in nonhomogeneous parabolic temperature distribution across the width. The creep deformations and stress relaxtions obtained are reasonable compared to those of other analytical approach [11]. The second example is stress analysis of a plane strain slotted solid propellant grain structure subjected to temperature variation during curing. The finite element solution for displacement agrees well but stress deviates from quasi-elastic solution which presented in White[6]. These two examples verify the applicability of the present model without any difficulty for thermoviscoelastic structures analysis.

DIFFERENTIAL GOVERNING EQUATIONS OF THE ENDOCHRONIC THERMOVISCOELASTIC FORMULATION

The endochronic constitutive equations for isotropic thermorheologically simple materials under small temperature variation and small deformation conditions are derived by Valanis [12] as follows:

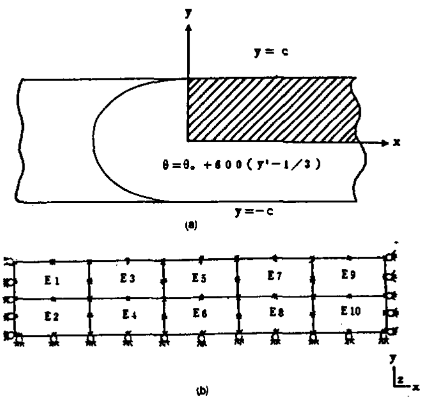

Fig. 1: Flat plate with nonuniform temperature distribution. (a) schematic interpretation and temperature function (b) finite element meshes and boundary conditions of one-fourth finite model.

$$S_{ij}(z) = 2\int_0^z \mu(\xi - \xi')\frac{\partial e_{ij}}{\partial z'}dz' \tag{2-1}$$

$$\sigma_{kk}(z) = 3\int_0^z K(\xi - \xi')\frac{\partial \epsilon_{kk}}{\partial z'}dz' + 3\int_0^z D(\xi - \xi')\frac{\partial \theta}{\partial z'}dz' \tag{2-2}$$

where S_{ij} and e_{ij} are deviatoric stress and strain tensors; σ_{kk} and ϵ_{kk} are hydrostatic stress and strain tensors; $\mu(\xi)$ and $K(\xi)$ are Shear Relaxation Modulus and Bulk Relaxation Modulus; $D(\xi)$ and θ are Temperature Effect Modulus and temperature increment respectively. The time hardening creep intrinsic time is defined with respect to effective stress σ_e and t as [13]

$$dz = g(\sigma_e, t)dt \tag{2-3}$$

The reduced time ξ is defined as

$$\xi = \int_0^z \phi_T[T(X_k, \bar{z}')]d\bar{z}' \tag{2-4}$$

where ϕ_T is the reciprocal of Time-Temperature Shift Factor a_T; $T(X_k, \bar{z})$ is a temperature function corresponding to postition. coordinate X_k and material time coordinate $\bar{z}$. The material functions $K(z)$, $D(z)$ and $\mu(z)$ are assumed in the following form

$$K(z) = H(z)K \tag{2-5}$$

$$D(z) = -3\alpha K \tag{2-6}$$

and

$$2\mu(z) = \mu_0 + \mu_1 e^{-\alpha_1 z} \tag{2-7}$$

where α, K and H(z) are thermal expansion coefficient, Bulk Constant and Step Function; μ_0, μ_1 and α_1 are material constants. Through the similar procedures proposed recently by the authors[13], the two term endochronic differential

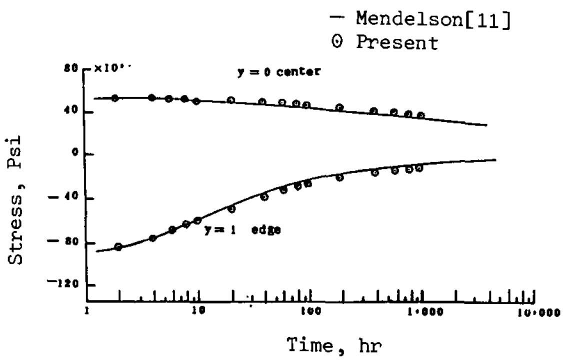

Fig. 2: Stresses relaxation in X-direction at center and outer edges with right end plate fixed.

thermoviscoelastic constitutive euqations can be written as

$$d\sigma_{ij} = 2\mu(o)d\epsilon_{ij} + [k - \frac{2}{3}\mu(o)]\delta_{ij}d\epsilon_{kk} + h_{ij}dz - 3\alpha K\delta_{ij}d\theta \qquad (2\text{-}8)$$

where $$h_{ij} = -\alpha_1\varphi(z)\int_0^z \mu_1 e^{-\alpha_1(\xi-\xi')}\frac{\partial e_{ij}}{\partial z'}dz' \qquad (2\text{-}9)$$

Equation (2-8) can be reduced to a matric form as

$$\{d\sigma\} = [c]\{d\varepsilon\} + \{dH_{ve}\} + \{dH_{TP}\} \qquad (2\text{-}10)$$

where [c] is elastic coefficient matrix, $\{dH_{ve}\}$ is viscoelastic differential stress, $\{dH_{TP}\}$ is temperature differential stress.

The differential finite element governing equilibrium equations are derived by using Eq. (2-10) and the principle of virtual work as follows:

$$[K]\{dq\} = \{dP_{ex}\} + \{dP_{ve}\} + \{dP_{TP}\} \qquad (2\text{-}11)$$

where

$$[K] = \int_v [B]^T[C][B]\,dv \qquad (2\text{-}12)$$

$$\{dP_{ex}\} = \int_v [N]^T\{d\bar{F}\}dv + \int_{s_\sigma}[N]^T\{d\bar{T}\}ds \qquad (2\text{-}13)$$

$$\{dP_{ve}\} = -\int_v [B]^T\{dH_{ve}\}dv \qquad (2\text{-}14)$$

$$\{dP_{TP}\} = -\int_v [B]^T\{dH_{TP}\}dv \qquad (2\text{-}15)$$

[N] is shape function matrix; [B] is strain-displacement coefficient matrix; $\{\bar{T}\}$ is surface traction force; $\{\bar{F}\}$ is unit volume body force.

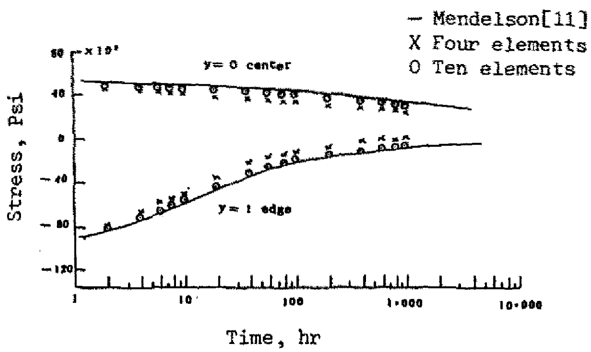

Fig. 3: Stresses relaxation in X-direction at center and outer edges with right end free.

NUMERICAL EXAMPLES AND RESULT DISCUSSION

To evaluate the capability of the endochronic thermoviscoelastic finite element algorithm, two examples are anlayzed and compared with analytic solutions or other finite element approaches.

The first problem shown in fig. 1(a) is an uniaxial(x-dir.) infinite plate subjected to a nonuniform parabolic temperature distributions $\theta = \theta_0 + 600(y^2 - \frac{1}{3})$, along y direction, where θ_0 is an arbitrary reference temperature. A qüarter of finite plate is used for analysis which is shown in fig. 1(b). The material parameters are adopted[11] as : $E = 28 \times 10^6, \alpha = 9.5 \times 10^{-6}$. If we choose $\nu = \mu_0 = 0$, then $K = \frac{E}{3}$, $\mu(0) = \frac{E}{2}$ and $\mu_1 = E$. In this material-temperature independent case $\varphi_T = 1$. The intrinsic time dz is defined as $dz = \sigma_{et}^A t^B dt$ where α_1, A and B are found by fitting the uniaxial creep test curve $\dot{\varepsilon}_{11}^c = 3 \times 10^{-24} \sigma_{11}^4$ as A = 3, B = 0 and $\alpha_1 = 1.386 \times 10^{24}$. Fig. 2 shows the x-direction stresses relaxation at y = 0, and y = 1. The results of the present analysis is slightly larger than the analytical solutions [11], because the right end of the plate is fixed. It is concluded that the analytic solutions are lowere bound. Fig. 3 shows the stresses relaxation of plate with free right end. The results are a little less than the analytic solutions, thus the analytic solution becomes the upper bound.

In the second example we analyze a plane strain slotted grain configuration where the temperature field varies with time and position. Fig. 4 shows the dimension and finite element meshes of one-quarter segment of the cross section in which the outer boundary is fixed.

Material properties are adopted from Ref. [6] as : $\mu(t) = 33.337 + 3360.448x e^{-2.4245}$ ksi, k = 100,000 ksi. $\alpha = 6 \times 10^{-5}$ 1/°F and $a_T = 10^{\frac{-K_1(\theta - \theta_0)}{K_2 - (\theta - \theta_0)}}$, where K_1 =

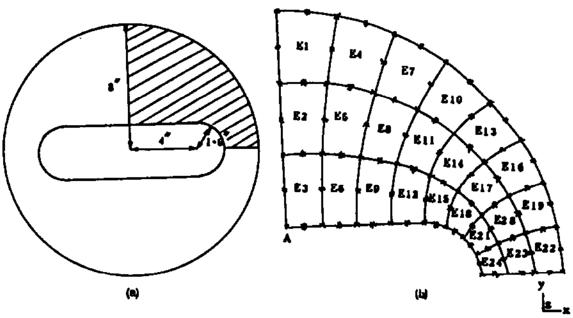

Fig. 4: Viscoelastic slotted grain (a) cross sectional dimension (b) finite element meshes of one-fourth model.

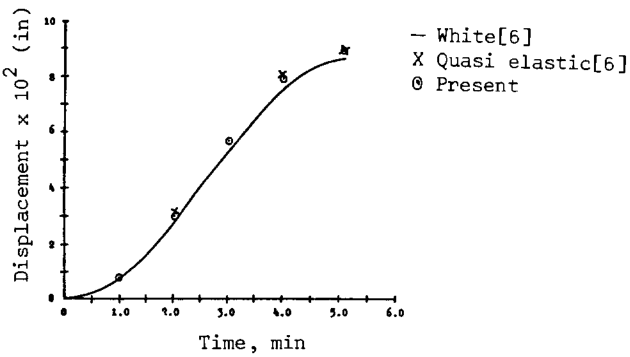

Fig 5: Displacement at point A.

3.05, K_2 = 225.7 and θ_0 = 70°F. The material constants μ_0 = 66.674, μ_1 = 6720.976 and α_1 = 2.4245 are determined.

In this example a temperature field $\theta(r,t) = 70 - 70(1-\cos\frac{\pi t}{5})(\frac{r}{8})^3$ °F is input.

The intrinsic time measure dz = dt and time increment Δt = 0.05 min. Fig. 5 is y-direction creep deformation at point A. The displacement curves obtained by using present model and by White [6] are a little higher than the quasi-elastic solutions which can be regarded as satisfactory. Fig. 6 shows the y-direction stresses varying with time at right slot tip. The stresses obtained by the present alaysis is below that of quasi-elastic analysis but the stresses obtained by White [6] is above that of quasi-elastic analysis.

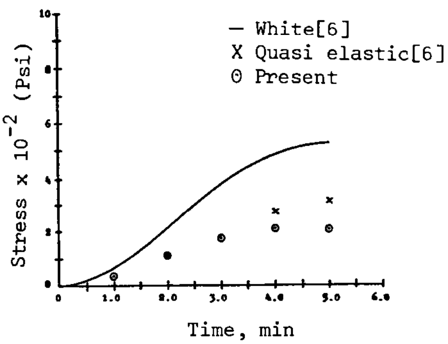

Fig. 6: Stress near right slot tip.

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